Algebra (बिजगणित)

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बीजगणितीय सूत्र :-

(i) $(x+y)^2 \Rightarrow x^2+y^2+2xy$

(ii) $(x-y)^2 \Rightarrow x^2+y^2-2xy$

(iii) $x^2-y^2 \Rightarrow (x+y)(x-y)$

(iv) $x^3-y^3 \Rightarrow (x-y)(x^2+xy+y^2)$

(v) $x^3+y^3 \Rightarrow (x+y)(x^2-xy+y^2)$

(vi) $(x+y)^3 \Rightarrow x^3+y^3+3xy(x+y)$

(vii) $(x-y)^3 \Rightarrow x^3-y^3-3xy(x-y)$

(viii) $(x+y+z)^2 \Rightarrow x^2+y^2+z^2+2(xy+yz+zx)$

(ix) $(x+y-z)^2 \Rightarrow x^2+y^2+z^2+2(xy-yz-zx)$

(x) $(x-y+z)^2 \Rightarrow x^2+y^2+z^2+2(-xy-yz+zx)$

(xi) $(x+y+z)^3 \Rightarrow \boxed{a^3+b^3+c^3+3(a+b)(b+c)(a+c)}$

$x^3+y^3+z^3+3(x+y)(y+z)(z+x)$

<u>When</u> $x+y+z=0$

<u>then</u> $x^3+y^3+z^3-3xyz=0$

$\boxed{x^3+y^3+z^3=3xyz}$

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$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$a+b+c=0$

then $\boxed{a^3+b^3+c^3=3abc}$

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Ex.1 $1570 \times 87 + 1570 \times 13$ का मान होगा।

Sol $\Rightarrow ab + ac \Rightarrow a(b+c)$

$\Rightarrow 1570(87+13) \Rightarrow 1570 \times 100 \Rightarrow 157000$ Ans.

Ex.2 $140 \times 140 + 60 \times 280 + 60 \times 60$ हल करो.

Sol $(a+b)^2 \Rightarrow a^2 + b^2 + 2ab$

$a = 140$
$b = 60$

$\Rightarrow (140+60)^2 = (200)^2 = 40000$ Ans.

Ex.3 $203 \times 203 - 197 \times 197$ का मान होगा।

Sol $a^2 - b^2 = (a+b)(a-b)$

$\Rightarrow (203+197)(203-197) \Rightarrow 400 \times 6 = 2400$ Ans.

Ex.4 $\dfrac{4.54 \times 4.54 \times 4.54 - 2.14 \times 2.14 \times 2.14}{4.54 \times 4.54 + 4.54 \times 2.14 + 2.14 \times 2.14}$ का मान

Sol $(a^3 - b^3) = (a-b)(a^2 + ab + b^2)$

$\Rightarrow \dfrac{(4.54)^3 - (2.14)^3}{(4.54)^2 + 4.54 \times 2.14 + (2.14)^2} \Rightarrow 4.54 - 2.14$

$= 2.40$ Ans

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Ex.5 $\dfrac{(9.64+7.36)^2-(9.64-7.36)^2}{(9.64\times 7.36)}$ का मान

Sol $(a+b)^2-(a-b)^2 = 4ab \Rightarrow$

$\Rightarrow \dfrac{4ab}{ab} = 4$ Ans.

Ex.6 $\dfrac{(0.337+0.126)^2-(0.337-0.126)^2}{0.337\times 0.126} = 4$ Ans.

Ex.7 $\dfrac{3.25\times 3.20-3.20\times 3.05}{0.064}$ का मान

Sol $\dfrac{3.20(3.25-3.05)}{0.064} \Rightarrow \dfrac{3.20\times 0.20}{0.064}$

$\Rightarrow \dfrac{320\times 20}{642} = 10$ Ans.

Ex.8 $\left\{\dfrac{(0.1)^2-(0.01)^2}{0.0001}+1\right\}$ का मान होगा।

$\because x^2-y^2 = (x+y)(x-y) \Rightarrow \dfrac{(0.1+0.01)(0.1-0.01)}{0.0001}+1$

$\Rightarrow \dfrac{(0.11)(0.09)}{0.0001}+1 \Rightarrow 11\times 9+1$

$= 99+1 = 100$ Ans.

Ex.9 $\dfrac{0.2\times0.2\times0.2 + 0.02\times0.02\times0.02}{0.6\times0.6\times0.6 + 0.06\times0.06\times0.06}$ का मान

Sol $\Rightarrow \dfrac{(0.2)^3+(0.02)^3}{(0.6)^3+(0.06)^3} \Rightarrow \dfrac{(0.2)^3+(0.02)^3}{((0.2)\times3)^3+((0.02)\times3)^3}$

$\Rightarrow \dfrac{1}{(3)^3}\left\{\dfrac{(0.2)^3+(0.02)^3}{(0.2)^3+(0.02)^3}\right\} = \dfrac{1}{3^3} = \dfrac{1}{27}$ Ans.

Ex.10 यदि $\dfrac{x}{y} = \dfrac{4}{3}$ हो तो $\dfrac{4x+3y}{4x-3y}$ का मान होगा ?

Sol I st $\dfrac{x}{y} = \dfrac{4}{3} \Rightarrow \dfrac{4\times4+3\times3}{4\times4-3\times3} = \dfrac{16+9}{16-9} \Rightarrow \dfrac{25}{7}$ Ans.

IInd $\dfrac{y\left(4\frac{x}{y}+3\right)}{y\left(4\frac{x}{y}-3\right)} \Rightarrow \dfrac{4\times\frac{4}{3}+3}{4\times\frac{4}{3}-3}$

$\Rightarrow \dfrac{\frac{16}{3}+\frac{3}{1}}{\frac{16}{3}-\frac{3}{1}} \quad \dfrac{\frac{16+9}{3}}{\frac{16-9}{3}} = \dfrac{25}{7}$ Ans.

Ex.11 यदि $x=19$ तथा $y=18$ हो तो $\dfrac{x^2+y^2+xy}{x^3-y^3}$ का मान

Sol $\dfrac{x^2+y^2+xy}{(x-y)(x^2+y^2+xy)} = \dfrac{1}{x-y} = \dfrac{1}{19-18} = 1$ Ans.

Ex.12 यदि $a+b+c=16$ तथा $ab+bc+ac=40$ हो तो $a^2+b^2+c^2=?$

Sol $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$(16)^2 = a^2+b^2+c^2+2(40)$

$256-80 = a^2+b^2+c^2 \Rightarrow 176$ Ans.

Ex.13 यदि $x-y=8$ तथा $xy=20$ हो तो x^3-y^3 का मान होगा.

Sol $\because x^3-y^3 = (x-y)(x^2+xy+y^2)$

$\because (x-y)^2 = x^2+y^2-2xy \Rightarrow x^2+y^2 = (x-y)^2+2xy$

$\Rightarrow x^2+y^2 = (8)^2+2\times 20 \Rightarrow 64+40 = 104$

$x^3-y^3 = (x-y)(x^2+xy+y^2)$

$\Rightarrow 8(104+20] \Rightarrow 124\times 8 = 992$ Ans.

Ex.14 $(999-96)^2-3\times 999+3\times 96$ बराबर $\frac{?}{?}$

Sol $(999-96)^2-3(999-96)$

$\Rightarrow (999-96)\{(999-96)-3\}$

$\Rightarrow (903)\{999-96-3\}$

$\Rightarrow (903)(999-99) \Rightarrow (903)\times 900$ Ans.

Ex.15 $\left(\sqrt{2}+\frac{1}{\sqrt{2}}\right)^2$ को सरल करने पर प्राप्त होगा ?

Sol $\because (x+y)^2 = x^2+y^2+2xy$

$\therefore \quad (\sqrt{2})^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + 2 \times \sqrt{2} \times \frac{1}{\sqrt{2}} \Rightarrow 2 + \frac{1}{2} + 2$

$\Rightarrow \frac{4+1+4}{2} = \frac{9}{2} \Rightarrow 4\frac{1}{2}$ Ans.

Ex.16 $\left(x+\frac{1}{x}\right) = 6$ हो तो $x^2+\frac{1}{x^2} = ?$

Sol $\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2\times x \times \frac{1}{x} = 36$

$x^2+\frac{1}{x^2}+2 = 36 \Rightarrow x^2+\frac{1}{x^2} = 36-2 = 34$ Ans.

Ex.17 $\left(x+\frac{1}{x}\right) = 5$ हो तो $x^4+\frac{1}{x^4}$ का मान होगा.

Sol $\left(x+\frac{1}{x}\right)^2 = 25 \Rightarrow x^2+\frac{1}{x^2}+2 = 25$

$x^2+\frac{1}{x^2} = 23$

$\left(x^2+\frac{1}{x^2}\right)^2 = (23)^2 \Rightarrow$

$x^4+\frac{1}{x^4}+2\times x^2 \times \frac{1}{x^2} = 529$

$\Rightarrow x^4+\frac{1}{x^4}+2 = 529$

$x^4+\frac{1}{x^4} = 529-2 = 527$ Ans.

Ex.18 $x^4 + \frac{1}{x^4} = 23$ हो तो $x + \frac{1}{x}$ का मान होगा ?

Sol $\left(x^2 + \frac{1}{x^2}\right)^2 \Rightarrow x^4 + \frac{1}{x^4} + 2$

$\left(x^2 + \frac{1}{x^2}\right)^2 = 23 + 2 \Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 25$

$\Rightarrow x^2 + \frac{1}{x^2} = 5 \Rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 5 + 2 \Rightarrow x + \frac{1}{x} = \sqrt{7}$ Ans.

Ex.19 यदि $x + \frac{1}{x} = 2\sqrt{3}$ हो तो $x^3 + \frac{1}{x^3} = ?$

Sol $\left(x^3 + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3 \cancel{x} \times \frac{1}{\cancel{x}} \left(x + \frac{1}{x}\right)$

$\Rightarrow \left(2\sqrt{3}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$

$8 \times 3\sqrt{3} = x^3 + \frac{1}{x^3} + 3 \times 2\sqrt{3}$

$\Rightarrow x^3 + \frac{1}{x^3} = 24\sqrt{3} - 6\sqrt{3} \Rightarrow 18\sqrt{3}$ Ans.

Ex.20 $x + \frac{1}{x} = 3$ हो तो $x^6 + \frac{1}{x^6}$ का मान होगा

Sol $\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$

$(3)^3 = x^3 + \frac{1}{x^3} + 3(3)$

$\Rightarrow 27 = x^3 + \frac{1}{x^3} + 9 \Rightarrow \left(x^3 + \frac{1}{x^3}\right)^2 = (18)^2$

$x^6 + \frac{1}{x^6} + 2\frac{\cancel{x^3}}{\cancel{x^3}} = 324 \Rightarrow x^6 + \frac{1}{x^6} = 324 - 2$

$= 322$ Ans.

Ex.21 यदि $x = \sqrt{3} + \sqrt{2}$ हो, तो $\left(x^3 + \frac{1}{x^3}\right)$ का मान

Sol $x = \sqrt{3} + \sqrt{2}$ then $\frac{1}{x} = \sqrt{3} - \sqrt{2}$

$\Rightarrow \left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right)$

$\Rightarrow \left(\sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(\sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2}\right)$

$(2\sqrt{3})^3 = x^3 + \frac{1}{x^3} + 3(2\sqrt{3})$

$8 \times 3 \times \sqrt{3} = x^3 + \frac{1}{x^3} + 6\sqrt{3} \Rightarrow x^3 + \frac{1}{x^3} = 24\sqrt{3} - 6\sqrt{3}$

$\Rightarrow 18\sqrt{3}$ Ans.

Ex.22 यदि $2x - \frac{1}{3x} = 4$ हो तो $27x^3 - \frac{1}{8x^3}$ का मान होगा

$\Rightarrow 2x \times \frac{3}{2x} - \frac{1}{3x \times \frac{3}{2}} = 4 \times \frac{3}{2}$

$\Rightarrow 3x - \frac{1}{2x} \Rightarrow 6 \Rightarrow \left(3x - \frac{1}{2x}\right)^3 = (3x)^3 + \left(\frac{1}{2x}\right)^3 - 3 \times \frac{3x}{2x}\left(3x - \frac{1}{2x}\right)$

$\Rightarrow (6)^3 = 27x^3 + \frac{1}{8x^3} - \frac{9}{2}[6]$

$216 = 27x^3 + \frac{1}{8x^3} - 9/2 \times 6$ 3

$27x^3 + \frac{1}{8x^3} = 216 + 27 \Rightarrow 243 = 243$ Ans

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Ex.28 यदि $x+\frac{1}{x}=3$ हो तो $x^5+\frac{1}{x^5}$ का मान होगा ?

Sol $x^2+\frac{1}{x^2}=(3)^2-2 \Rightarrow 9-2=7$ -①

$x^3+\frac{1}{x^3}=(3)^3-3\times3 \Rightarrow 27-9=18$ -②

① × ② $\left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)=7\times18$

$x^2\times x^3+x^2/x^3+x^3/x^2+\frac{1}{x^2\times x^3}=126$

$x^5+\boxed{\frac{1}{x}+x}+\frac{1}{x^5}=126 \Rightarrow x^5+\frac{1}{x^5}=126-3=123$ Ans.
3

Ex.29 यदि $\sqrt{x}+\frac{1}{\sqrt{x}}=\sqrt{5}$ हो तो $x^2+\frac{1}{x^2}$ का मान होगा ?

Sol $\left(x+\frac{1}{x}\right) \Rightarrow (5)^2-2=25-3=23$

$\Downarrow$

$\left(x^2+\frac{1}{x^2}\right)=(23)^2-2=529-2=527$ Ans.

Ex.30 यदि $\left(x+\frac{1}{x}\right)^2=3$ हो तो x^3+1/x^3 का मान होगा।

Sol $\left(x+\frac{1}{x}\right)^2=3 \Rightarrow x+\frac{1}{x}=\sqrt{3}$

$\Rightarrow x^3+\frac{1}{x^3} \Rightarrow (\sqrt{3})^3-3\times\sqrt{3} \Rightarrow 3\sqrt{3}-3\sqrt{3}=0$ Ans.

Ex.31 यदि $2p+\frac{1}{p}=4$ हो, तो $p^3+\frac{1}{8p^3}$ बराबर होगा।

Sol $2p+\frac{1}{p}=4 \Rightarrow 2\left(p+\frac{1}{2p}\right)=4 \Rightarrow p+\frac{1}{2p}=2$

~~$\left(p^3+\frac{1}{8p^3}\right)=(2)^3-3\times2\Rightarrow8-6=2$~~

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$\left(p+\frac{1}{2p}\right)^3=p^3+\frac{1}{8p^3}+3\times\frac{1}{2p}\left(p+\frac{1}{2p}\right)$

$(2)^3=p^3+\frac{1}{8p^3}+\frac{3}{2}\times2 \Rightarrow 8-3=5$ Ans.

Ex.32 $2x + \frac{2}{x} = 1$ हो तो, $x^3 + \frac{1}{x^3}$ का मान होगा।

Sol $2\left(x + \frac{1}{x}\right) = 1 \Rightarrow x + \frac{1}{x} = \frac{1}{2}$

$\Rightarrow x^3 + \frac{1}{x^3} \Rightarrow \left(\frac{1}{2}\right)^3 - 3 \times \frac{1}{2} \Rightarrow \frac{1}{8} - \frac{3}{2} \Rightarrow \frac{1-12}{8} = -\frac{11}{8}$ Ans.

Ex.33 यदि $x = (\sqrt{2}-1)^{1/2}$ हो तो $x^2 - \frac{1}{x^2}$ का मान होगा।

Sol $x^2 = \left((\sqrt{2}-1)^{1/2}\right)^2 \Rightarrow \sqrt{2}-1$

$\frac{1}{x^2} = \frac{1}{(\sqrt{2}-1)^2} = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1$

$x^2 + \frac{1}{x^2} = \sqrt{2} - 1 + \sqrt{2} + 1$

$2\sqrt{2}$ Ans.

Ex.33 यदि $\frac{x}{y} = \frac{a+2}{a-2}$ हो तो $\frac{x^2-y^2}{x^2+y^2}$ का मान होगा।

Sol $\frac{y^2\left(\frac{x^2}{y^2} - 1\right)}{y^2\left(x^2/y^2 + 1\right)} \Rightarrow \frac{\left(\frac{a+2}{a-2}\right)^2 - 1}{\left(\frac{a+2}{a-2}\right)^2 + 1} \Rightarrow \frac{\frac{(a+2)^2}{(a-2)^2} - 1}{\frac{(a+2)^2}{(a-2)^2} + 1}$

$\Rightarrow \frac{\frac{(a+2)^2 - (a-2)^2}{(a-2)^2}}{\frac{(a+2)^2 + (a-2)^2}{(a-2)^2}} \Rightarrow \frac{a^2 + 4 + 4a - a^2 + 4 - 4a}{a^2 + 4 + 4a + a^2 + 4 - 4a} = \frac{8a}{2a^2 + 8}$

$\Rightarrow \frac{8a}{2(a^2+4)} \Rightarrow \frac{4a}{a^2+4}$ Ans.